Government of **Western Australia**
Curriculum Council

WACE
Western Australian Certificate of Education

Western Australian Certificate of Education
Examination, 2011

Question/Answer Booklet

MATHEMATICS: SPECIALIST 3C/3D

Section Two: Calculator-assumed

Please place your student identification label in this box

Student Number: In figures | | | | | | | | |

In words _____

_____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time for this section: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators satisfying the conditions set by the Curriculum Council for this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

 Ref: 11-140

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	40	
Section Two: Calculator-assumed	13	13	100	80	
			Total	120	100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2011.* Sitting this examination implies that you agree to abide by these rules.

2. Answer the questions according to the following instructions.

   Section Two: Write answers in this Question/Answer Booklet. Answer **all** questions.

   Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

   It is recommended that you **do not use pencil**, except in diagrams.

3. You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

See next page

Section Two: Calculator-assumed **(80 Marks)**

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

Working time: 100 minutes.

Question 8 **(5 marks)**

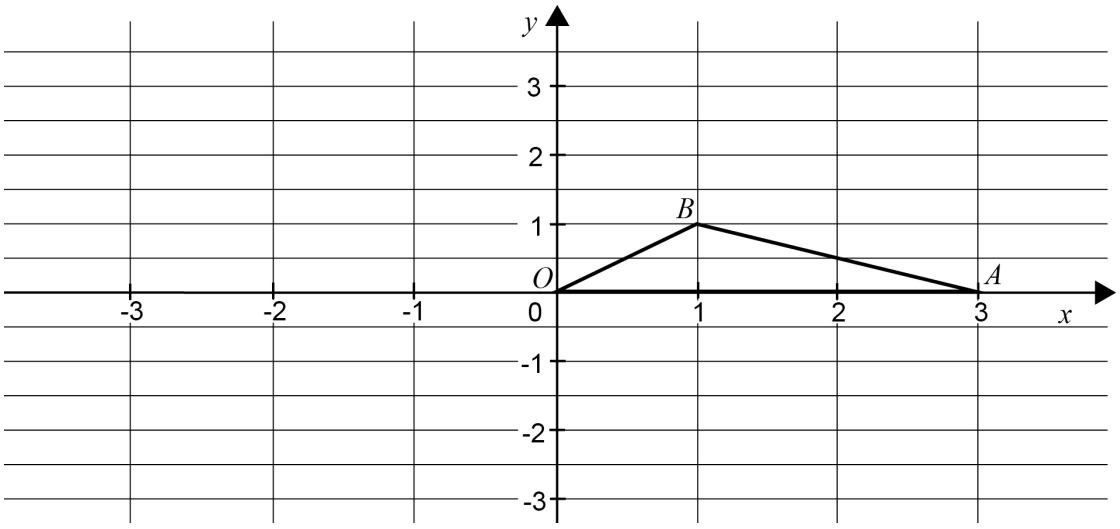
Radium decays at a rate proportional to its present mass; that is, if $Q(t)$ is the mass of radium present at time *t*, then $\frac{dQ}{dt} = kQ$.

It takes 1600 years for any mass of radium to reduce by half.

(a) Find the value of k. (3 marks)

(b) A factory site is contaminated with radium. The mass of radium on the site is currently five times the safe level. How many years will it be before the mass of radium reaches the safe level? (2 marks)

Question 9 **(4 marks)**



A triangle has vertices $O(0, 0)$, $A(3, 0)$ and $B(1, 1)$, as shown in the diagram above.

(a) Write down the matrix that rotates triangle OAB through 90° clockwise about the origin. (1 mark)

(b) If triangle OAB is transformed by a dilation about the origin of scale factor k ($k > 0$), determine the matrix that will create an image of area 24 square units. (3 marks)

See next page

Question 10 **(8 marks)**

Two radio-controlled model planes take off at the same time from two different positions and with constant velocities. Model A leaves from the point with position vector $(-3\boldsymbol{i} - 7\boldsymbol{j})$ metres and has velocity $(5\boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k})$ m/s; model B leaves from the point with position vector $(7\boldsymbol{i} - \boldsymbol{j} - 8\boldsymbol{k})$ metres and has velocity $(3\boldsymbol{i} - 4\boldsymbol{j} + 6\boldsymbol{k})$ m/s.

(a) Find the distance between the two model planes after 1 second of flight. (3 marks)

(b) Find: (5 marks)

(i) the shortest distance between the two model planes.

(ii) the time when this occurs.

See next page

Question 11 **(4 marks)**

The triangle ABC has vertices $A(2, 1, 0)$, $B(3, -3, 3)$ and $C(5, 0, 4)$.

(a) Find the size of $\angle ABC$ correct to the nearest degree. (2 marks)

(b) Given that the vector $(-13\boldsymbol{i} + 5\boldsymbol{j} + 11\boldsymbol{k})$ is perpendicular to the plane which contains the triangle ABC, find the vector equation of this plane. (2 marks)

See next page

Question 12 **(6 marks)**

Three dry-cleaning companies, A, B and C compete for business. Each year A loses 40% of its customers to B and 20% to C; B loses 30% to A and 50% to C; C loses 60% to A and 10% to B.

(a) Complete the following transition matrix. (2 marks)

		From A	From B	From C
To	A	0.4	____	____
	B	0.4	____	____
	C	0.2	____	____

(b) At the end of 2011, company A will have 80% of market share, while B and C will have 10% each. What will be the market share of each company at the end of 2012?
(2 marks)

(c) If these conditions remain unchanged, what will be the long-term percentage market share for each company, correct to **one (1)** decimal place? (2 marks)

See next page

Question 13 **(6 marks)**

An engine piston undergoes simple harmonic motion that can be described by the differential equation $\frac{d^2x}{dt^2} = -9x$, where x m is the displacement of the piston from its mean position at t seconds.

(a) Write down the period of the motion. (1 mark)

(b) If the maximum speed of the piston is 5 m/s, find the amplitude of the motion.

(2 marks)

See next page

Question 13 (continued)

(c) The amplitude and period of the motion are now changed, but the piston still undergoes simple harmonic motion. These new readings are taken:

when $x = 1$ m, speed $= \sqrt{60}$ m/s; when $x = 3$ m, speed $= \sqrt{28}$ m/s

Find the new exact values for: (3 marks)

(i) the period.

(ii) the amplitude.

Question 14 **(5 marks)**

The points P, Q and R are such that $\overrightarrow{PQ} = 5\boldsymbol{i}$ and $\overrightarrow{PR} = \boldsymbol{i} + 4\boldsymbol{j} + 2\boldsymbol{k}$.

Find the vector $\overrightarrow{RM}$ that is parallel to $\overrightarrow{PQ}$ and such that the size of $\angle RQM$ is 90º.

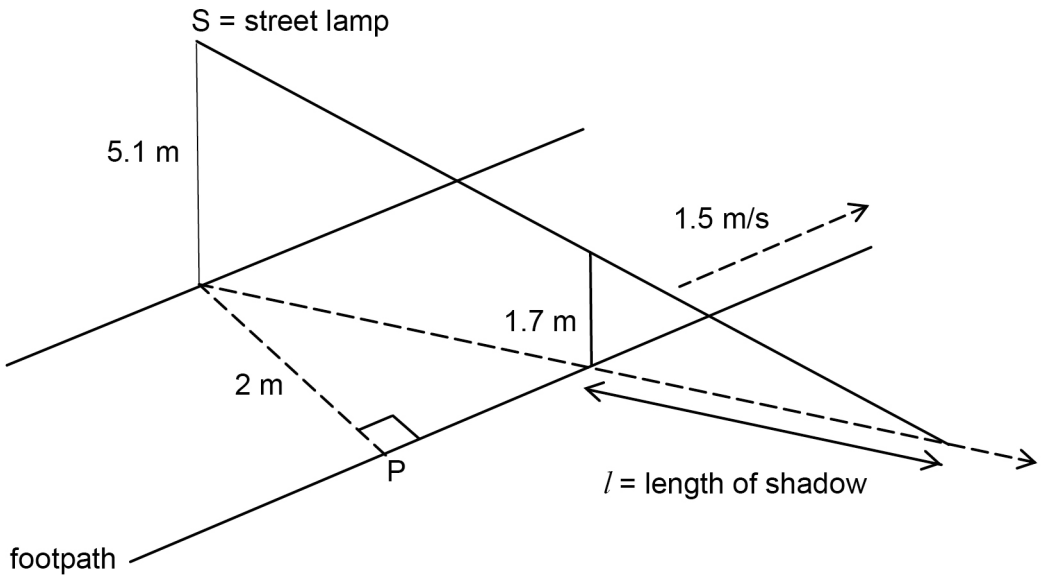
See next page

Question 15 **(5 marks)**

(a) Use Euler's formula $\left(e^{ix} = \cos x + i \sin x\right)$ to show that $\sin x = \dfrac{e^{ix} - e^{-ix}}{2i}$. (3 marks)

(b) Expand $\left(\dfrac{e^{ix} - e^{-ix}}{2i} \right)^5$ to obtain an expression for $\sin^5 x$ in terms of $\sin x$, $\sin 3x$ and $\sin 5x$. (2 marks)

Question 16 **(6 marks)**



In the diagram above, P is the initial position of a boy, of height 1.7 metres, who is walking along a straight footpath in the direction shown.

S is the position of a street lamp of height of 5.1 metres; its base is 2 metres from P.

The street lamp will cast a moving shadow of the boy as he continues to walk along the footpath at 1.5 m/s.

See next page

Question 16 (continued)

(a) If x metres is the distance walked by the boy, show that the length (l metres) of the boy's shadow is $l = \frac{1}{2}\sqrt{4 + x^2}$. (3 marks)

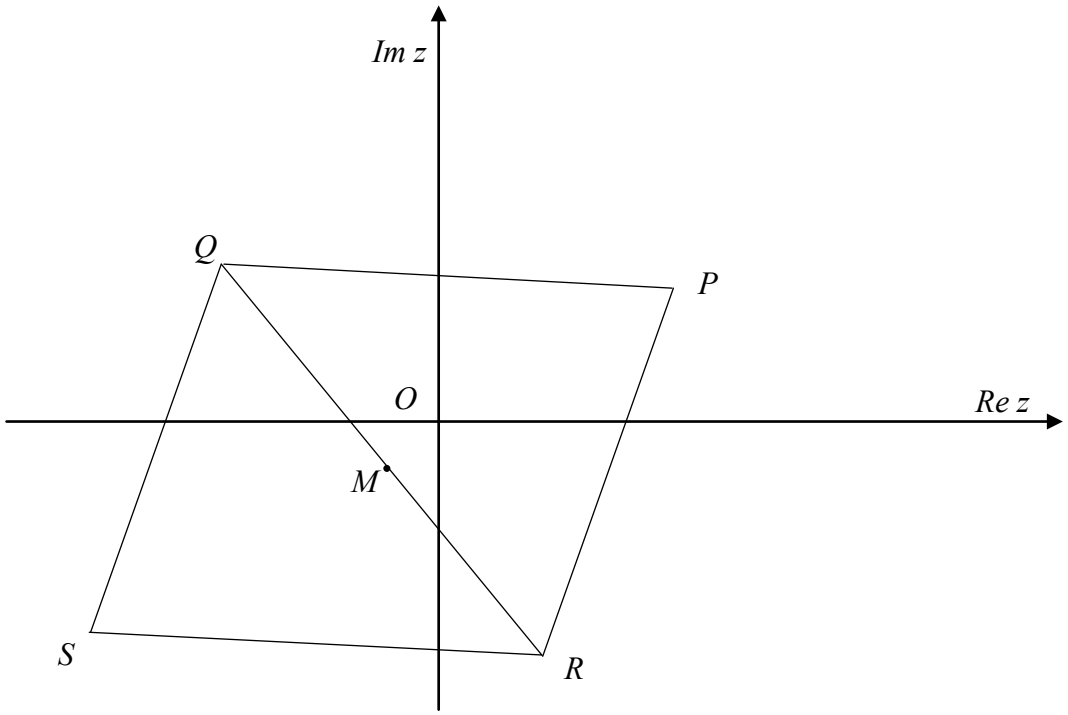
(b) Find the rate of change, in m/s, of the length of the boy's shadow after 5 seconds.

(3 marks)

See next page

Question 17 **(9 marks)**

The point P on the Argand diagram below represents the complex number z. The points Q and R represent the points wz and $\overline{w}z$ respectively, where $w = \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}$. The point M is the midpoint of QR. (The diagram is not drawn to scale.)



(a) If $z = r\,cis(\theta)$, find wz and $\overline{w}z$ in polar form. (2 marks)

Question 17 (continued)

(b) Hence explain why $\left|\overrightarrow{OP}\right| = \left|\overrightarrow{OQ}\right| = \left|\overrightarrow{OR}\right|$. (2 marks)

(c) Show that the complex number representing M is $-\frac{1}{2}z$. (2 marks)

(d) The point S is chosen so that $PQSR$ is a parallelogram. Find the complex number represented by S in terms of z. (3 marks)

See next page

Question 18 **(8 marks)**

A model for a population, P, of numbats is

$P = \dfrac{900}{3 + 2e^{-t/4}}$, where *t* is the time in years from today.

(a) What is the population today? (1 mark)

(b) What does the model predict that the eventual population will be? (1 mark)

See next page

Question 18 (continued)

(c) By first expressing $e^{-t/4}$ in terms of P, or otherwise, show that P satisfies the differential equation $\frac{dP}{dt} = \frac{P}{4}\left(1 - \frac{P}{300}\right)$. (4 marks)

(d) What is the instantaneous percentage annual rate of growth today? (2 marks)

See next page

Question 19 **(7 marks)**

Let $f(n) = 3^{n+2} + (-1)^n \times 2^n$, for all positive integers n.

(a) Show that $2f(n+1) - f(n)$ is divisible by 5. (2 marks)

See next page

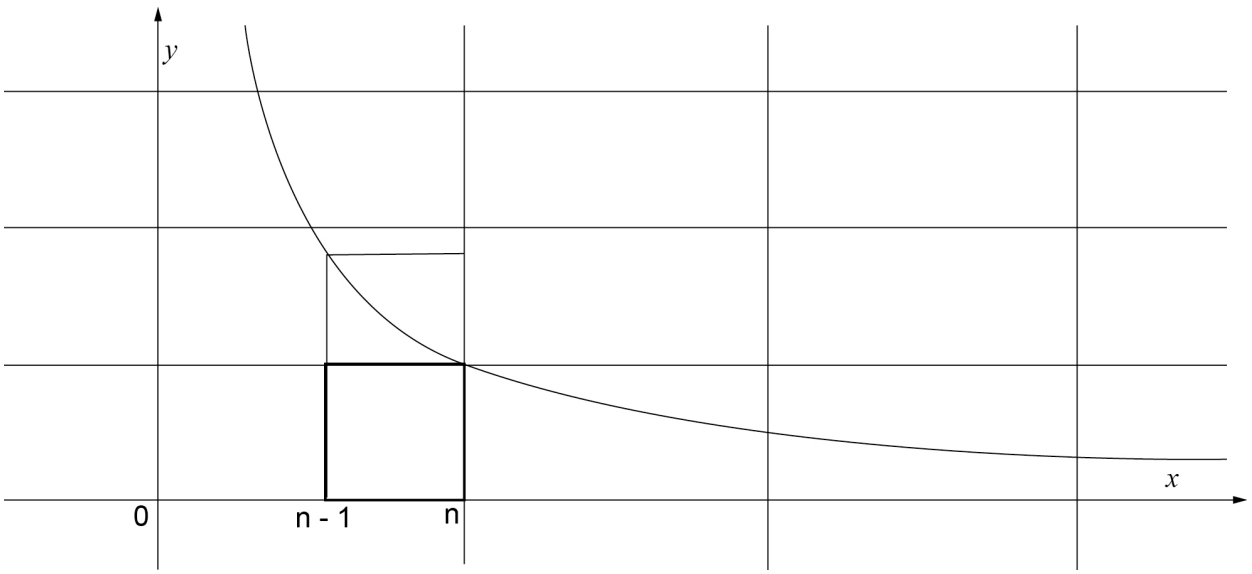
Question 19 (continued)

(b) Hence, or otherwise, prove by induction that $f(n)$ is divisible by 5. (5 marks)

See next page

Question 20 **(7 marks)**

Let n be a positive integer greater than 1. The area of the region under the curve $y = \frac{1}{x}$ from $x = n - 1$ to $x = n$ lies between the areas of the two rectangles, as shown in the diagram.



(a) Use the diagram to show that $e^{-\frac{n}{n-1}} < \left(1 - \frac{1}{n}\right)^n < e^{-1}$. (6 marks)

Question 20 (continued)

(b) Hence deduce $\lim_{n\to\infty}\left(1-\frac{1}{n}\right)^n$. (1 mark)

End of questions

Additional working space

Question number: _____

Additional working space

Question number: _____

Published by the Curriculum Council of Western Australia
27 Walters Drive
OSBORNE PARK WA 6017